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HEAT AND MASS TRANSFER AND PHYSICAL GASDYNAMICS

Dissociated Gas Flow in the Boundary Layer Along Bodies of Revolution of a Porous Contour

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Abstract—This paper studies the ideally dissociated gas flow in the so-called frozen boundary layer on bodies of revolution. The contour of the body of revolution is porous. The governing boundary layer equations are brought to a generalized form. The obtained equations are numerically solved using the finite differences method. Based on the obtained solutions, distributions of physical quantities in the boundary layer are presented in the form of diagrams. Conclusions on behaviour of these quantities are also made.

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INTRODUCTION

This paper investigates the ideally dissociated gas (air) flow in the boundary layer on bodies of revolution. The contour of the body within the fluid is porous. The dissociated gas flows in the conditions of "frozen" thermochemical activity (the second boundary case of dissociated gas flow) [1, 2].

This is a continuation of our earlier investigations conducted as a part of a scientific project financed by the Ministry of Science of the Republic of Serbia.

The objective of the paper is to apply the general similarity method to the studied flow problem.

The general similarity method was first introduced by L.G. Loitsianskii [3] for solution of boundary layer equations. Later, this method was improved by V.N. Saljnikov [4]—Saljnikov's version. Investigators of St. Petersburg and Belgrade schools of boundary layer used this method to solve numerous boundary layer flow problems. Significant results were achieved in application of this method to incompressible fluid flow, to **MHD** and temperature boundary layers for both non-porous and porous contour of the body within the fluid [3, 5, 6]. The original version of the general similarity method was successfully applied to the planar boundary layer in the conditions of the so-called equilibrium dissociation [7, 8]. Saljnikov's version of this method was used for solution of numerous planar problems of both dissociated [9] and ionized gas flow [10] in the boundary layer.

The results presented here are obtained by application of Saljnikov's version of the general similarity method. Unlike other methods [11], the application of this improved method involves usage of the momentum equation and sets of similarity parameters. The momentum equation is derived from the continuity equation and the corresponding dynamic equation.

GOVERNING EQUATIONS. MOMENTUM EQUATION

When gas flows at high velocities (e.g. supersonic flight of an aircraft through the Earth's atmosphere), the temperature in the viscous boundary layer increases significantly. High temperatures cause thermochemical reactions of dissociation of atoms in a molecule and a reverse reaction of their recombination. Therefore, the gas (air) becomes a multicomponent mixture of atomic and molecular components of the same gas. When the rates of dissociation and recombination are high enough, thermochemical equilibrium is established in this area of the boundary layer [1, 7, 8]. However, when the rates of the thermoschemical reactions are low enough, atoms are not formed at the expense of molecules. In this second boundary case of the ideally dissociated gas flow, chemical activity is frozen but the change in the concentration of the atomic component is conditioned by the diffusion process [1, 12].

If we exclude the pressure from the equations for the laminar and steady boundary layer of the ideally dissociated gas (air), then the equation system of the "frozen" boundary layer on bodies of revolution [12–14] has the following form:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v r^j) = 0, \quad (j = 1),$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right),$$

$$\rho u \frac{\partial \alpha}{\partial x} + \rho v \frac{\partial \alpha}{\partial y} = \frac{\partial}{\partial y}\left(\rho D \frac{\partial \alpha}{\partial y}\right),$$

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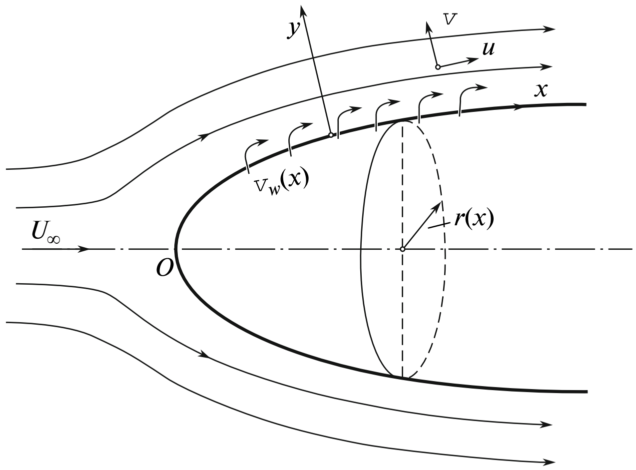

Fig. 1. Gas flow along the body of revolution.

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-u\rho_e u_e\frac{du_e}{dx} \quad (1)$$
$$+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\rho D(c_{pA}-c_{pM})\frac{\partial\alpha}{\partial y}\frac{\partial T}{\partial y},$$
$$u=0,\ v=v_w(x),\ T=T_w,\ \alpha=\alpha_w \text{ for } y=0,$$
$$u\to u_e(x),\ T\to T_e(x),\ \alpha\to\alpha_e(x) \text{ for } y\to\infty.$$

The equation system (1) represents the mathematical model of this problem. The first equation of this system is a continuity equation of the axisymmetrical flow of the ideally dissociated gas on bodies of revolution ($j=1$). The second one is a dynamic equation, while the third one is an equation of atomic component diffusion. The atom mass concentration is $C_A=\rho_A/\rho=\alpha$. The fourth equation of the system is an energy equation, while the fifth one represents a state equation of the ideally dissociated gas. Note that the equation system of the "frozen" boundary layer for the planar steady flow [9] differs from the given system (1) only in the continuity equation. In this paper $j=0$ and the equation does not contain the radius $r(x)$ of the cross-section of the body of revolution.

The equation of the atomic component diffusion in the system (1) contains only a mass diffusion coefficient of the atomic component D on the right hand-side. This equation has been derived using a simplified Fick's law in which barodiffusion and thermodiffusion are neglected [1].

The diffusion flow along the main flow direction is known to be negligibly small. At higher supersonic speed, the temperature distribution in the boundary layer is basically determined mainly by the influence of viscosity.

For the physical quantities [1, 12], we use the notation common in the boundary layer theory. Here, $u(x,y)$ is a longitudinal projection of the velocity in the boundary layer, $v(x, y)$ —transversal projection, ρ—density of the ideally dissociated gas (mixture), μ—dynamic viscosity, T—absolute temperature, λ—coefficient of the thermal conductivity, D—coefficient of the atomic component diffusion, c_p —specific heat capacity of the ideally dissociated gas at constant pressure, k—Boltzman constant, m—mass and R—gas constant. The subscript "e" stands for the physical quantities at the outer edge of the boundary layer, the subscript "w" stands for the conditions on the wall of the body of revolution, while the subscripts A and M stand for the atomic and molecular component of the ideally dissociated gas.

The radius $r(x)$ of the cross-section of the body of revolution is normal to the axis of revolution. The function $r(x)$ practically defines the contour of the body of revolution (Fig. 1). Here, $v_w(x)$ denotes the given velocity at which the dissociated gas flows normally through the solid porous wall ($v_w>0$ or $v_w<0$). It is assumed that the boundary layer thickness $\delta(x)$ is everywhere considerably smaller than the radius of the body of revolution ($\delta(x)\ll r(x)$). Therefore, this thickness can be neglected compared to $r(x)$ [14]. This assumption does not apply to long thin bodies [15].

The general similarity method is based on the usage of the momentum equation. During the transformation of the momentum equation of the axisymmetrical flow problem, it was shown [16] that the continuity equation of compressible fluid should be written in a more suitable form as:

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j\right]=0,\quad (j=1). \quad (2)$$

In this equation, L is a characteristic constant length, and for the numerical calculation [8], we can consider it to be: $L=1$. The Eq. (2) for the axisymmetrical flow ($j=1$) with $L=\text{const}$ comes down to the first equation of the system (1).

By the usual procedure [16], taking the boundary conditions into consideration, from the continuity Eq. (2) and the dynamic equation of the system (1), by integration transversally to the boundary layer (from $y=0$ to $y\to\infty$), we obtain the equation

$$\frac{d}{dx}\left[\int_0^\infty\rho u\left(\frac{r}{L}\right)^j(u_e-u)dy\right]-\rho_w v_w\left(\frac{r}{L}\right)^j u_e$$
$$=\left(\frac{r}{L}\right)^j\frac{du_e}{dx}\int_0^\infty(\rho u-\rho_e u_e)dy+\left(\frac{r}{L}\right)^j\left(\mu\frac{\partial u}{\partial y}\right)_{y=0}.$$

As with other compressible fluid flow problems, for solution of the integrals, instead of the physical coordinates x, y, new variables are introduced in the form of the following transformations:

$$s(x)=\frac{1}{\rho_0\mu_0}\int_0^x\rho_w\mu_w\left(\frac{r}{L}\right)^{2j}dx,$$
$$z(x,y)=\frac{1}{\rho_0}\left(\frac{r}{L}\right)^j\int_0^y\rho dy,\quad (j=1). \quad (3)$$

In the transformations (3), ρ_0 and $\mu_0 = \rho_0\nu_0$ denote the known values of the density and dynamic viscosity of the dissociated gas at a certain point of the boundary layer ν_0-kinematic viscosity). Here ρ_w and μ_w are the given values of these quantities on the wall of the body of revolution. These transformations for $j = 0$ were used in [7] but for the case of a non-porous wall of the body within the fluid ($v = v_w = 0$).

Having changed the variables, the momentum equation is obtained rather easily, especially since in the course of solution of the integrals, the variable $z(x, y)$ changes only because of the change of the coordinate y (for any x). This equation is presented in its three forms:

$$\frac{dZ^{**}}{ds} = \frac{F^*}{u_e}, \quad \frac{df}{ds} = \frac{u_e'}{u_e}F^* + \frac{u_e''}{u_e'}f, \qquad \frac{\Delta^{**\prime}}{\Delta^{**}} = \frac{F^*}{2f}\frac{u_e'}{u_e}, \tag{4}$$

where the prime denotes a derivative with respect to the longitudinal variable s. Note that these three forms of the Eq. (4) are formally the same and the forms of the momentum equation obtained for incompressible fluid.

In order to obtain the momentum equation, conditional displacement thickness $\Delta^*(s)$, conditional momentum loss thickness $\Delta^{**}(s)$, nondimensional friction function $\zeta(s)$ and parameter of the form $f(s)$ have been defined by the following expressions:

$$\Delta^*(s) = \int_0^\infty\left(\frac{\rho_e}{\rho} - \frac{u}{u_e}\right)dz,$$

$$\Delta^{**}(s) = \int_0^\infty\frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)dz, \quad \frac{\Delta^*}{\Delta^{**}} = H, \tag{5}$$

$$\zeta(s) = \left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0},$$

$$Z^{**} = \frac{\Delta^{**2}}{\nu_0}, \quad f(s) = \frac{u_e'\Delta^{**2}}{\nu_0} = u_e'Z^{**} = f_1(s).$$

The characteristic function of the boundary layer F^* in the obtained momentum Eq. (4) is determined by the expression

$$F^* = 2[\zeta - (2 + H)f] + \frac{2}{(r/L)^j}\frac{v_w\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w}, \quad (j = 1).$$

For the case of a flow along a nonporous wall ($v_w = 0$), this function formally comes down to the corresponding function ($F^* = F$) of the planar dissociated gas flow [7], i.e., to the known expression [3] which corresponds to the incompressible fluid flow. For $j = 0$ the function is identical to the function F_{dp} for the planar flow of the equilibrium-dissociated gas [17] along a porous wall.

When according to the second term of the expression for F^*, we define the porosity parameter as

$$\Lambda(s) = -\frac{1}{(r/L)^j}\frac{\mu_0}{\mu_w}v_w\frac{\Delta^{**}}{\nu_0} = -V_w\frac{\Delta^{**}}{\nu_0} = -\frac{V_w}{\sqrt{\nu_0}}Z^{**1/2} = \Lambda_1(s), \tag{6}$$

$$V_w = \frac{1}{(r/L)^j}\frac{\mu_0}{\mu_w}v_w;$$

the characteristic function F^* of the boundary layer can be written in the form of the expression

$$F^* = 2[\zeta - (2 + H)f] - 2\Lambda. \tag{7}$$

The expression (7) will be further used in this paper for numerical solution of the boundary layer equations. The quantity $V_w(s)$ represents a conditional velocity of the fluid injection.

TRANSFORMATIONS OF THE GOVERNING EQUATIONS

The stream function $\psi(s, z)$ is introduced in accordance with the relations

$$u = \frac{\partial\psi}{\partial z}, \qquad \tilde{v} = \frac{1}{(r/L)^{2j}}\frac{\rho_0\mu_0}{\rho_w\mu_w}\left[u\frac{\partial z}{\partial x} + v\frac{\rho}{\rho_0}\left(\frac{r}{L}\right)^j\right] = -\frac{\partial\psi}{\partial s}, \quad (j = 1); \tag{8}$$

that follow from the continuity Eq. (2). For $j = 0$ the relations (8) come down to the corresponding expressions used in the papers [7, 9] for the planar flow of dissociated gas.

Applying the new transformations (3) and introducing the stream function by the relations (8), the governing boundary layer equation system (1) is brought to this form:

$$\frac{\partial\psi}{\partial z}\frac{\partial^2\psi}{\partial s\partial z} - \frac{\partial\psi}{\partial s}\frac{\partial^2\psi}{\partial z^2} = \frac{\rho_e}{\rho}u_e\frac{du_e}{ds} + \nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\psi}{\partial z^2}\right),$$

$$c_p\left(\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial s} - \frac{\partial\psi}{\partial s}\frac{\partial T}{\partial z}\right) = -\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial\psi}{\partial z} + \nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Pr}}c_p\frac{\partial T}{\partial z}\right) + \nu_0 Q\left(\frac{\partial^2\psi}{\partial z^2}\right)^2 + \nu_0(c_{pA} - c_{pM})\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial z}\frac{\partial T}{\partial z},$$

$$\frac{\partial\psi}{\partial z}\frac{\partial\alpha}{\partial s} - \frac{\partial\psi}{\partial s}\frac{\partial\alpha}{\partial z} = \nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial z}\right); \tag{9}$$

$$\frac{\partial\psi}{\partial z} = 0, \quad \frac{\partial\psi}{\partial s} = -\tilde{v}_w = -\frac{\mu_0}{\mu_w}v_w\frac{1}{(r/L)^j} = -V_w,$$

$$T = T_w, \quad \alpha = \alpha_w \quad \text{for} \quad z = 0,$$

$$\frac{\partial\psi}{\partial z} \to u_e(s), \quad T \to T_e(s), \quad \alpha \to \alpha_e(s) \quad \text{for} \quad z \to \infty.$$

The nondimensional function Q, Prandtl and Schmidt numbers are defined by the expressions:

$$Q(s,z) = \frac{\rho\mu}{\rho_w\mu_w}, \quad \mathrm{Pr} = \frac{\mu c_p}{\lambda}, \quad \mathrm{Sm} = \frac{\mu}{\rho D}. \tag{10}$$

Since the boundary condition on the porous wall in the system (9) is $\partial\psi/\partial s = -V_w \neq 0$, it can be brought down to zero, which is the case for the nonporous wall. Then, the stream function $\psi(s,z)$ is divided into two parts in the form of the expressions

$$\psi(s,z) = \psi_w(s) + \overline{\psi}(s,z), \quad \overline{\psi}(s,0) = 0; \tag{11}$$

where $\psi_w(s)$ denotes the stream function along the body within the fluid ($\psi_w(s) = \psi(s,0)$).

Applying the introduced transformations (3), (8) and (11), the governing equation system (1), i.e. (9) is brought to:

$$\frac{\partial\overline{\psi}}{\partial z}\frac{\partial^2\overline{\psi}}{\partial s\partial z} - \frac{\partial\overline{\psi}}{\partial s}\frac{\partial^2\overline{\psi}}{\partial z^2} - \frac{d\psi_w}{ds}\frac{\partial^2\overline{\psi}}{\partial z^2}$$
$$= \frac{\rho_e}{\rho}u_e\frac{du_e}{ds} + \nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\overline{\psi}}{\partial z^2}\right),$$
$$c_p\left(\frac{\partial\overline{\psi}}{\partial z}\frac{\partial T}{\partial s} - \frac{\partial\overline{\psi}}{\partial s}\frac{\partial T}{\partial z}\right) - c_p\frac{d\psi_w}{ds}\frac{\partial T}{\partial z}$$
$$= -\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial\overline{\psi}}{\partial z} + \nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Pr}}c_p\frac{\partial T}{\partial z}\right)$$
$$+ \nu_0 Q\left(\frac{\partial^2\overline{\psi}}{\partial z^2}\right)^2 + \nu_0(c_{pA} - c_{pM})\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial z}\frac{\partial T}{\partial z}, \tag{12}$$
$$\frac{\partial\overline{\psi}}{\partial z}\frac{\partial\alpha}{\partial s} - \frac{\partial\overline{\psi}}{\partial s}\frac{\partial\alpha}{\partial z} - \frac{d\psi_w}{ds}\frac{\partial\alpha}{\partial z} = \nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial z}\right),$$
$$\overline{\psi}(s,z) = 0, \quad \frac{\partial\overline{\psi}}{\partial z} = 0, \quad T = T_w,$$
$$\alpha = \alpha_w \quad \text{for} \quad z = 0,$$
$$\frac{\partial\overline{\psi}}{\partial z} \to u_e(s), \quad T \to T_e(s),$$
$$\alpha \to \alpha_e(s) \quad \text{for} \quad z \to \infty.$$

Each of the equations of the system (12) contains a new term (unlike the system (9)) in which $d\psi_w/ds$ appears. This derivative is determined by the expression

$$\frac{d\psi_w}{ds} = \frac{d\psi(s,0)}{ds}$$
$$= -\frac{\mu_0}{\mu_w}v_w\frac{1}{(r/L)^j} = -V_w, \quad (j = 1). \tag{13}$$

For $j = 0$, the obtained equation system (12) is identical to the corresponding equation system obtained in [9] for the planar flow of dissociated gas along a porous wall—which is expected.

In order for the general similarity method to be applied, this boundary condition should be brought to zero. This will be explained shortly.

GENERALIZED BOUNDARY LAYER EQUATION SYSTEM

Further application of Saljnikov's version of the general similarity method [4, 12] involves another transformation of the variables in the system (12):

$$s = s(x), \quad \eta(s,z) = \frac{u_e^{b/2}}{S(s)}z,$$
$$S(s) = \left(a\nu_0\int_0^s u_e^{b-1}ds\right)^{1/2}, \quad a, b = \text{const}; \tag{14}$$
$$\overline{\psi}(s,z) = u_e^{1-b/2}S(s)\Phi(s,\eta),$$
$$T(s,z) = T_1\overline{T}(s,\eta); \quad T_1 = \text{const}.$$

In the expressions (14): $\eta(s,z)$ —denotes a newly introduced transversal variable, $\Phi(s,\eta)$ —nondimensional stream function, $\overline{T}$ —nondimensional temperature, T_1 —temperature at a forward stagnation point of the body of revolution, while a and b —stand for real constants.

Based on (14) important quantities and characteristics of the boundary layer can be expressed in the form of more suitable relations as:

$$S(s) = \frac{u_e^{b/2}}{B(s)}\Delta^{**}(s), \quad \eta(s,z) = \frac{B(s)}{\Delta^{**}(s)}z,$$
$$H = \frac{\Delta^*(s)}{\Delta^{**}(s)} = \frac{A}{B}, \quad \zeta = B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0},$$
$$\overline{\psi}(s,z) = \frac{u_e(s)\Delta^{**}(s)}{B(s)}\Phi(s,\eta), \tag{15}$$
$$A(s) = \int_0^\infty\left(\frac{\rho_e}{\rho} - \frac{\partial\Phi}{\partial\eta}\right)d\eta,$$
$$B(s) = \int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1 - \frac{\partial\Phi}{\partial\eta}\right)d\eta, \quad \frac{f}{B^2} = \frac{au_e'}{u_e^b}\int_0^s u_e^{b-1}ds;$$

where A and B are considered to be continual functions of the variable s.

Using the newly introduced variables (14), the equation system (12) is, after complicated and comprehensive transformations, brought down to a form suitable for further analysis. Each of thus obtained equations contains the term (u_e/u_e'), while the energy equation explicitly contains the term u_e^2/T_1. The solution of the obtained system will depend on each concrete form of the law of this velocity. Therefore, the obtained system is not generalized in terms of Loitsianskii [3, 4]. The analysis has shown that from the very beginning the appropriate sets of parameters should be introduced into the transformations (14). Hence, in order to apply the general similarity method in Saljnikov's version, the functions $\Phi, \overline{T}$ and α are introduced by the expressions:

$$\begin{gathered}\bar{\psi}(s,z) = \frac{u_e\Delta^{**}}{B}\Phi\left[(\eta;\, \kappa,\, (f_k),\, (\Lambda_k)\right],\\ (k = 1,2,3,\ldots),\\ T(s,z) = T_1\bar{T}\left[(\eta;\, \kappa,\, (f_k),\, (\Lambda_k)\right],\\ \alpha = \alpha\left[(\eta;\, \kappa,\, (f_k),\, (\Lambda_k)\right];\end{gathered} \tag{16}$$

where $\kappa = f_0$ is a local compressibility parameter [7].

In the so-called similarity transformations (16), the set of parameters $f_k(s)$ of Loitsianskii type [3] and the set of parameters $\Lambda_k(s)$ of the porous wall [16] represent new independent variables and they are defined as:

$$\begin{gathered}\kappa(s) = f_0(s) = \frac{u_e^2}{2c_{p1}T_1}, \quad f_k(s) = u_e^{k-1}u_e^{(k)}Z^{**k},\\ \Lambda_k(s) = -u_e^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)} Z^{**k-1/2}.\end{gathered} \tag{17}$$

The introduced sets of parameters satisfy the recurrent simple differential equations:

$$\begin{gathered}\frac{u_e}{u_e'}f_1\frac{d\kappa}{ds} = 2\kappa f_1 = \theta_0,\\ \frac{u_e}{u_e'}f_1\,\frac{df_k}{ds} = \left[(k-1)f_1 + kF^*\right]f_k + f_{k+1} = \theta_k,\\ \frac{u_e}{u_e'}f_1\frac{d\Lambda_k}{ds} = \left\{(k-1)f_1 + \left[(2k-1)/2\right]F^*\right\}\Lambda_k\\ + \Lambda_{k+1} = \chi_k, \quad (k = 1,2,3,\ldots).\end{gathered} \tag{18}$$

The structure of the expressions (17) and (18) is the same as the structure of the corresponding expressions for the case of incompressible fluid flow [3].

Having applied the similarity transformations (14) and (16), the equation system (12) with the boundary conditions is finally transformed into the following system:

$$\begin{gathered}\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}\\ + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] + \frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}\\ = \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_k} - \frac{\partial\Phi}{\partial f_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right.\\ \left. + \sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right],\\ \frac{\partial}{\partial\eta}\left[\frac{Q}{\Pr}\frac{c_p}{c_{p1}}\frac{\partial\bar{T}}{\partial\eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\frac{c_p}{c_{p1}}\Phi\frac{\partial\bar{T}}{\partial\eta}\\ - \frac{\rho_e}{\rho}\frac{2\kappa f_1}{B^2}\frac{\partial\Phi}{\partial\eta} + 2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2\\ + \frac{Q}{\mathrm{Sm}}\frac{c_{pA} - c_{pM}}{c_{p1}}\frac{\partial\alpha}{\partial\eta}\frac{\partial\bar{T}}{\partial\eta} + \frac{c_p}{c_{p1}}\frac{\Lambda_1}{B}\frac{\partial\bar{T}}{\partial\eta}\\ = \frac{1}{B^2}\frac{c_p}{c_{p1}}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{T}}{\partial f_k} - \frac{\partial\Phi}{\partial f_k}\frac{\partial\bar{T}}{\partial\eta}\right)\right.\\ \left. + \sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{T}}{\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\bar{T}}{\partial\eta}\right)\right],\\ \frac{\partial}{\partial\eta}\left[\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial\eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta} + \frac{\Lambda_1}{B}\frac{\partial\alpha}{\partial\eta}\\ = \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial f_k} - \frac{\partial\Phi}{\partial f_k}\frac{\partial\alpha}{\partial\eta}\right)\right.\\ \left. + \sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\alpha}{\partial\eta}\right)\right];\\ \Phi = 0, \quad \frac{\partial\Phi}{\partial\eta} = 0, \quad \bar{T} = \bar{T}_w,\\ \alpha = \alpha_w \quad \text{for} \quad \eta = 0,\\ \frac{\partial\Phi}{\partial\eta} \to 1, \quad \bar{T} \to \bar{T}_e = 1 - \kappa,\\ \alpha \to \alpha_w \quad \text{for} \quad \eta \to \infty.\end{gathered} \tag{19}$$

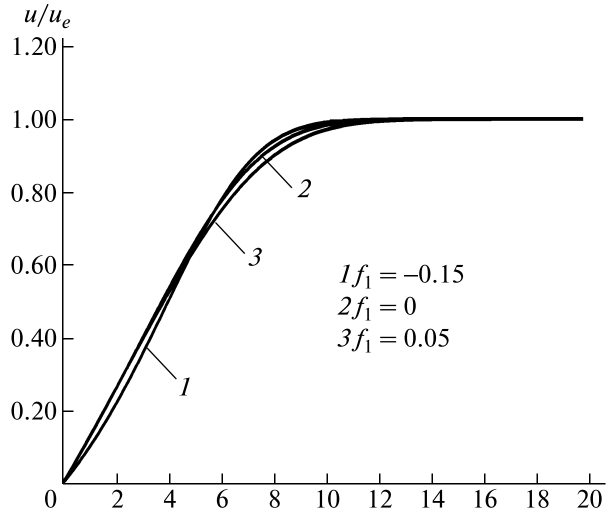

Fig. 2. Diagram of the nondimensional velocity $u/u_e\eta$ $f_0 = 0.10$, $\Lambda_1 = 0.05$, $T_1 = 4000$ K, $\bar{T}_w = 0.35$, $\alpha_w = 0.00$.

Since the distribution of the outer velocity $u_e(s)$ does not figure explicitly in the system (19), it is considered generalized ("universal"). Once again, note that the equation system obtained here has the same form as the equation system obtained for the dissociated gas planar flow along a porous wall [9]. For $j = 0$ these equation systems are identical. For the case of a nonporous wall $((v_w = 0 \Rightarrow V_w = 0 \Rightarrow \Lambda_k = 0))$ the equation system (19) is formally the same as the corresponding one obtained in [12].

Since the numerical solution of the system (19) is practically impossible, the number of parameters is limited, as expected in the boundary layer theory.

Then, the system is solved in the so-called n-parametric approximation.

The expressions for the density ratios ρ_e/ρ and ρ/ρ_w, that figure in the system (19) and in the function Q (10), are easily found from the state equation of the ideally dissociated gas (1) as:

$$\frac{\rho_e}{\rho} = \frac{1+\alpha}{1+\alpha_e}\frac{\overline{T}}{\overline{T}_e} = \frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-\kappa}, \quad \frac{\rho}{\rho_w} = \frac{1+\alpha_w}{1+\alpha}\frac{\overline{T}_w}{\overline{T}}, \tag{20}$$

with the condition [12] that $\alpha_e = \alpha_1$ and $c_{pe} = c_{p1}$. The viscosity ratio μ/μ_w of the ideally dissociated air can be determined using Fay–Riddell formula. This way, for the nondimensional function $Q(\alpha, \overline{T})$ we get the expression [9, 12]

$$Q = (1+\alpha)^{-3/2}(1+\alpha_w)\frac{\overline{T}_w}{\overline{T}}\Pi(\overline{T}), \tag{21}$$

where

$$\Pi(\overline{T}) = \left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{300}\right)^{3/2}\frac{413}{\frac{\overline{T}}{1-\kappa}T_1 + 113} + 3.7\left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{10000}\right)^2 - 2.35\left(\frac{\overline{T}}{1-\kappa}\frac{T_1}{10000}\right)^4. \tag{22}$$

For the specific heat capacity of the ideally dissociated air and for difference of specific heats of atomic and molecular components, the following expressions [12] are used:

$$\frac{c_p}{c_{p1}} = \frac{C^*(\alpha, \overline{T})}{C_1^*(\alpha_1)}, \quad \frac{c_{pA} - c_{pM}}{c_{p1}} = \frac{D^*(\overline{T})}{C_1^*(\alpha_1)}, \tag{23}$$

where the nondimensional functions C^*, C_1^* and D^* are determined with the relations:

$$C^*(\alpha, \overline{T}) = \frac{10}{7}\alpha + (1-\alpha)\left[1 + \frac{2}{7}e^{-(\overline{T}_v/\overline{T})^2}\right],$$
$$C_1^*(\alpha_1) = \frac{10}{7}\alpha_1 + (1-\alpha_1)\left[1 + \frac{2}{7}e^{-(\overline{T}_v)^2}\right],$$
$$D^*(\overline{T}) = \frac{3}{7} - \frac{2}{7}e^{-(\overline{T}_v/\overline{T})^2}, \quad \overline{T}_v = \frac{T_v}{T_1} = \frac{800}{T_1}, \tag{24}$$
$$c_{p1} = c_{p\infty}C_1^*, \quad c_{p\infty} = \frac{7}{2}R_M.$$

In the given relations, α_1 denotes the atomic component concentration at a forward stagnation point of the body of revolution, and c_{p1} denotes the specific heat capacity at the same point.

In the three-parametric ($f_0 = \kappa \neq 0$, $f_1 \neq 0$, $\Lambda_1 \neq 0$; $f_2 = f_3 = \ldots = \Lambda_2 = \Lambda_3 = \ldots = 0$), twice localized approximation ($(\partial/\partial\kappa = 0, \partial/\partial\Lambda_1 = 0)$) the equation system (19) is significantly simplified. Taking the expressions (20) and (23) into consideration, it comes down to:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1}\frac{\overline{T}}{1-\kappa} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] + \frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2} = \frac{F^*f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_1} - \frac{\partial\Phi}{\partial f_1}\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left[\frac{Q}{\Pr}\frac{C^*}{C_1^*}\frac{\partial\overline{T}}{\partial\eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial\overline{T}}{\partial\eta} - \frac{2\kappa\overline{T}}{1-\kappa}\frac{f_1}{B^2}\frac{1+\alpha}{1+\alpha_1}\frac{\partial\Phi}{\partial\eta} + 2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2 + \frac{Q}{\mathrm{Sm}}\frac{D^*}{C_1^*}\frac{\partial\alpha}{\partial\eta}\frac{\partial\overline{T}}{\partial\eta} + \frac{C^*}{C_1^*}\frac{\Lambda_1}{B}\frac{\partial\overline{T}}{\partial\eta} = \frac{C^*}{C_1^*}\frac{F^*f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\overline{T}}{\partial f_1} - \frac{\partial\Phi}{\partial f_1}\frac{\partial\overline{T}}{\partial\eta}\right), \tag{25}$$

$$\frac{\partial}{\partial\eta}\left[\frac{Q}{\mathrm{Sm}}\frac{\partial\alpha}{\partial\eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial\alpha}{\partial\eta} + \frac{\Lambda_1}{B}\frac{\partial\alpha}{\partial\eta} = \frac{F^*f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\alpha}{\partial f_1} - \frac{\partial\Phi}{\partial f_1}\frac{\partial\alpha}{\partial\eta}\right);$$

$$\Phi = 0, \quad \frac{\partial\Phi}{\partial\eta} = 0, \quad \overline{T} = \overline{T}_w = \text{const},$$
$$\alpha = \alpha_w = \text{const} \quad \text{for} \quad \eta = 0,$$
$$\frac{\partial\Phi}{\partial\eta} \to 1, \quad \overline{T} \to \overline{T}_e = 1-\kappa,$$
$$\alpha \to \alpha_e = \alpha_1 = \text{const} \quad \text{for} \quad \eta \to \infty.$$

The system of approximate generalized Eqs. (25) represents a mathematical model of the ideally dissociated gas (air) flow in the so-called frozen boundary layer along a porous wall on bodies of revolution. The parameter Λ_1 is characteristic for a porous wall. The parameters $\kappa = f_0$ and Λ_1, due to performed localization, play a role of simple parameters and they are given for numerical integration of the equation system (25).

Note that the obtained system of generalized Eqs. (25) is formally the same as the one obtained in [9] but for the planar boundary layer of the dissociated air. Clearly, this is due to the introduced transformations (3).

Investigations carried out for the dissociated gas (air) flow in the boundary layer [7, 12], showed that the influence of the compressibility parameter κ on distributions of the nondimensional velocity u/u_e and the concentration of atoms α is negligible. In these investigations, due to complex mathematical nature, all the derivatives with the respect to the parameter κ and the porosity parameter Λ_1 are neglected, as customary.

NUMERICAL SOLUTION

For numerical solution of the obtained equation system the order of the dynamic equation is first decreased by introduction of the change

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi(\eta, \kappa, f_1, \Lambda_1). \tag{26}$$

Now the equation system for numerical integration can be written as:

$$\begin{gathered}
\frac{\partial}{\partial \eta}\left(Q\frac{\partial \varphi}{\partial \eta}\right) + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial \varphi}{\partial \eta} \\
+ \frac{f_1}{B^2}\left[\frac{1+\alpha}{1+\alpha_1}\frac{\bar{T}}{1-\kappa} - \varphi^2\right] + \frac{\Lambda_1}{B}\frac{\partial \varphi}{\partial \eta} \\
= \frac{F^* f_1}{B^2}\left(\varphi\frac{\partial \varphi}{\partial f_1} - \frac{\partial \Phi}{\partial f_1}\frac{\partial \varphi}{\partial \eta}\right), \\
\frac{\partial}{\partial \eta}\left[\frac{Q}{\Pr}\frac{C^*}{C_1^*}\frac{\partial \bar{T}}{\partial \eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\frac{C^*}{C_1^*}\Phi\frac{\partial \bar{T}}{\partial \eta} \\
- \frac{2\kappa \bar{T}}{1-\kappa}\frac{f_1}{B^2}\frac{1+\alpha}{1+\alpha_1}\varphi + 2\kappa Q\left(\frac{\partial \varphi}{\partial \eta}\right)^2 + \frac{Q}{\mathrm{Sm}}\frac{D^*}{C_1^*}\frac{\partial \alpha}{\partial \eta}\frac{\partial \bar{T}}{\partial \eta} \\
+ \frac{C^*}{C_1^*}\frac{\Lambda_1}{B}\frac{\partial \bar{T}}{\partial \eta} = \frac{C^*}{C_1^*}\frac{F^* f_1}{B^2}\left(\varphi\frac{\partial \bar{T}}{\partial f_1} - \frac{\partial \Phi}{\partial f_1}\frac{\partial \bar{T}}{\partial \eta}\right), \\
\frac{\partial}{\partial \eta}\left[\frac{Q}{\mathrm{Sm}}\frac{\partial \alpha}{\partial \eta}\right] + \frac{aB^2 + (2-b)f_1}{2B^2}\Phi\frac{\partial \alpha}{\partial \eta} + \frac{\Lambda_1}{B}\frac{\partial \alpha}{\partial \eta} \\
= \frac{F^* f_1}{B^2}\left(\varphi\frac{\partial \alpha}{\partial f_1} - \frac{\partial \Phi}{\partial f_1}\frac{\partial \alpha}{\partial \eta}\right); \\
\varphi = 0, \quad \Phi = 0, \quad \bar{T} = \bar{T}_w = \text{const}, \\
\alpha = \alpha_w = \text{const} \quad \text{for} \quad \eta = 0, \\
\varphi \to 1, \quad \bar{T} \to \bar{T}_e = 1-\kappa, \\
\alpha \to \alpha_e = \alpha_1 = \text{const} \quad \text{for} \quad \eta \to \infty.
\end{gathered} \tag{27}$$

The system of partial differential conjugated generalized Eqs. (27) is numerically solved by the finite differences method—using the “passage method”. The values of the functions φ, Φ, $\bar{T}$ and α are calculated at discrete points of each calculating layer of the planar integration grid. Because of the complexity of the equation system, the number of discrete points for each calculating layer was $M = N = 401$.

For a concrete solution of the generalized equation system (27), i.e., of the corresponding system expressed by means of finite differences, a program in *FORTRAN* programming language was written. It is based on the program used in the paper [4]. When it comes to dissociated gas flow, Pr and Sm numbers are believed to be changing only little in the boundary layer [1]. In the paper [7], it is also stated that when the temperature is up to $T \approx 9000$ K, Pr and Le numbers depend on the temperature only insignificantly, therefore they can be considered constant. Furthermore, the investigations [12], carried out using the thermodynamic tables for air, have unambiguously shown that Pr number of dissociated gas can be considered approximately constant. In this paper, the equations have been solved with $\Pr = 0.712$. Since in the literature the “median” value of Le number for air is Le = 1.4, based on the relation $\mathrm{Sm} \cdot \mathrm{Le} = \Pr$ it can be concluded that Sm = 0.509. Therefore, the calculation is performed for specified value of Sm number. Standard values are taken for the constants a and b [4]: $a = 0.4408$ and $b = 5.7140$. The accepted values for the characteristic functions B and F^* at a zero iteration [4] are $B_{K+1}^0 = 0.469$ and $F_{K+1}^{*0} = 0.4411$.

RESULTS

The generalized equation system (27) was numerically solved for some cross-sections of the boundary layer. The results were obtained in the form of tables. The diagrams of physical quantities and characteristic functions of the boundary layer based on the specified tables were drawn for different values of the input parameters.

Only some of important diagrams are shown in this paper. Figure 2 shows the diagram of the nondimensional velocity u/u_e for three cross-sections of the boundary layer chosen at random. Figure 3 gives the diagram of the nondimensional temperature $\bar{T}$ for three cross-sections of the boundary layer, where the porosity parameter is $\kappa = f_0 = 0.2$. Figure 4 shows distribution of the nondimensional temperature $\bar{T}$ but for $\kappa = f_0 = 0.4$. In order to illustrate the influence of the compressibility parameter κ, a diagram of the nondimensional temperature for five different values of this parameter is presented (Fig. 5). Figure 6 presents the diagram of the atomic component concentration of the dissociated gas. Finally, Fig. 7 gives the distribution of the nondimensional friction function $\zeta(f_1)$ for a few values of the porosity parameter Λ_1.

CONCLUSIONS

This paper has shown that the general similarity method can be successfully applied to the studied flow problem, which was the primary objective of the paper. There are, however, some difficulties in application of this method, mainly of mathematical nature.

Compressible fluid flow is very complicated but the application of the general similarity method has given some quality results that illustrate the behaviour of distributions of the physical and characteristic quantities at certain cross-sections of the boundary layer.

While writing this paper, no experimental results that could be used for comparison were available to the authors; consequently the obtained results could be compared only with the results in the paper [7]. Based on the diagrams presented here and others ones not shown, it has been determined that the obtained solutions and figures of certain physical quantities have the

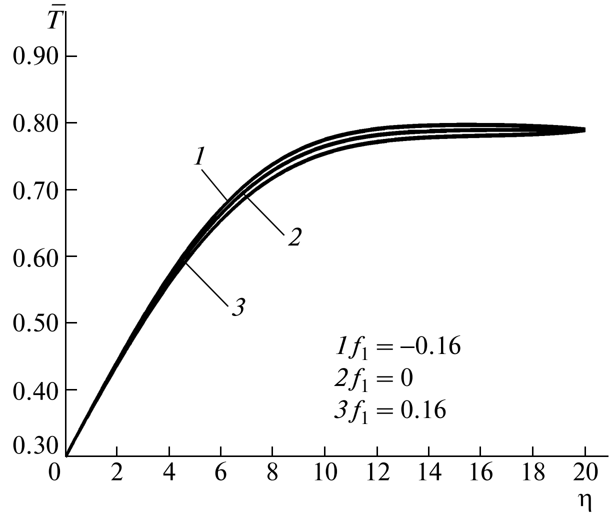

Fig. 3. Diagram of the nondimensional temperature $\bar{T}$. $f_0 = 0.20$, $\Lambda_1 = 0.10$, $T_1 = 5000$ K, $\bar{T}_w = 0.35$, $\alpha_w = 0.08$.

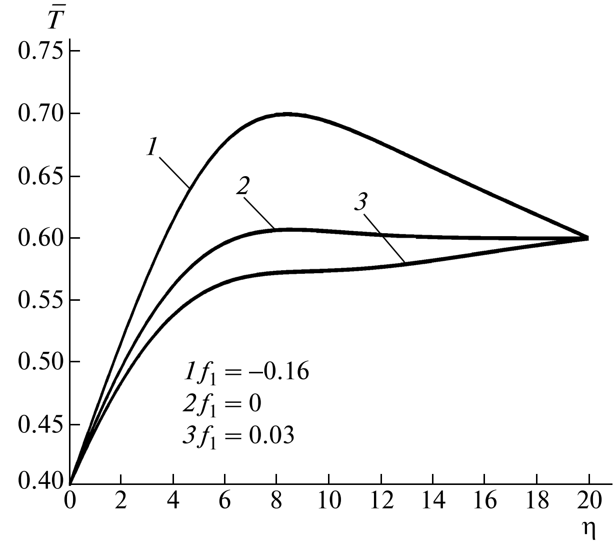

Fig. 4. Diagram of the nondimensional temperature $\bar{T}$ for different values of the parameter f_1. $f_0 = 0.40$, $\Lambda_1 = 0.10$, $T_1 = 4000$ K, $\bar{T}_w = 0.40$, $\alpha_w = 0.00$.

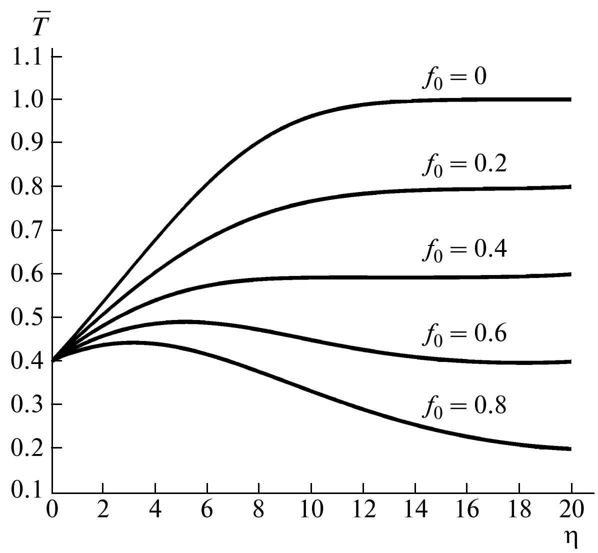

Fig. 5. Diagram of the nondimensional temperature $\bar{T}$ for different values of the parameter $\kappa = f_0$. $f_1 = 0.10$, $\Lambda_1 = 0.05$, $T_1 = 5000$ K, $\bar{T}_w = 0.40$, $\alpha_w = 0.08$.

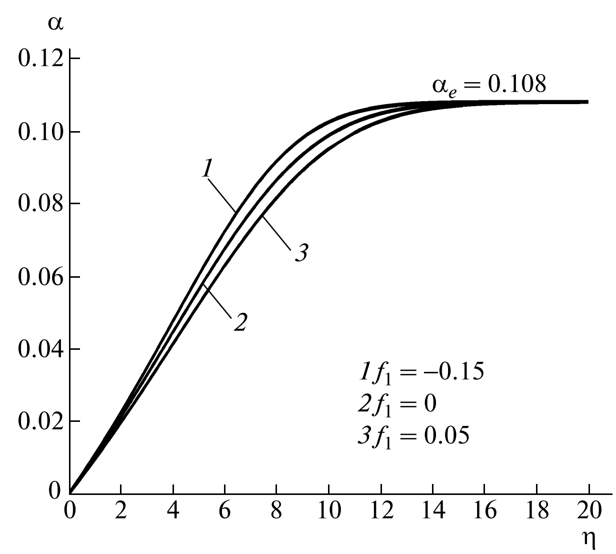

Fig. 6. Diagram of the atomic component concentration α. $f_0 = 0.10$, $\Lambda_1 = 0.05$, $T_1 = 4000$ K, $\bar{T}_w = 0.35$, $\alpha_w = 0.00$.

same behaviour as with problems of dissociated gas flow [7].

For the studied case of the ideally dissociated air flow in the boundary layer along a porous contour on bodies of revolution, the following concrete conclusions can be made:

• The nondimensional flow velocity u/u_e at different cross-sections of the boundary layer on bodies of revolution (for different f_1) converges very fast towards unity (Fig. 2).

• The atomic component concentration α also converges very fast towards the value at the outer edge of the boundary layer (Fig. 6).

• As with other problems of the dissociated air flow in the boundary layer, the nondimensional $\bar{T}$ has a characteristic behaviour. Namely, the curve of the nondimensional temperature at certain boundary layer cross-sections reaches a maximum value at the

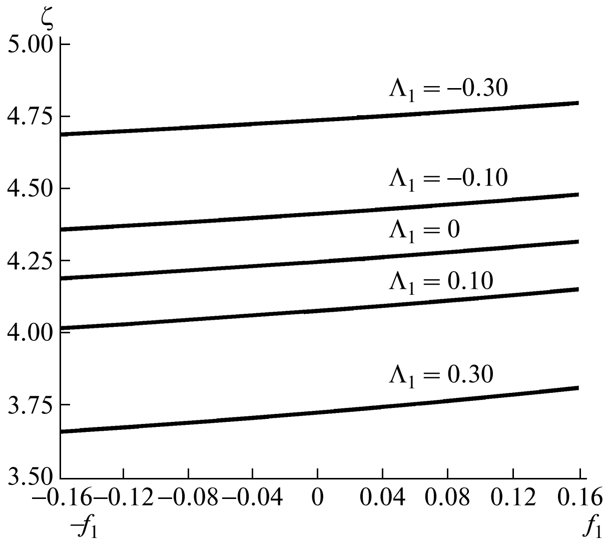

Fig. 7. Distribution of the nondimensional friction function $\zeta(f_1)$. $f_0 = 0.02$, $T_1 = 5000$ K, $\bar{T}_w = 0.35$, $\alpha_w = 0.08$.

outer edge of the boundary layer (Fig. 3; $\kappa_0 = 0.20$). The nondimensional temperature for $\kappa_0 = 0.40$ (Fig. 4, curve *1*) reaches its peak at the boundary layer and it surpasses the value of the temperature at the outer edge of the boundary layer ($\bar{T}_e = 1 - \kappa$). Based on these and the diagrams shown in Fig. 5, we can conclude that the compressibility parameter has a great influence on the nondimensional temperature of the boundary layer thus changing the general character of behaviour of this temperature. Therefore, the generalized boundary layer Eqs. (19) should be solved without localization per this parameter, which is even more difficult in terms of mathematics.

• The porosity parameter Λ_1 has a great influence on all the characteristics of the boundary layer (A, B, F*). The diagram in Fig. 7 clearly shows that this parameter has a significant influence on the nondimensional friction function ζ. As result, the values of this parameter, i.e. the values of the injection velocity v_w, have a significant influence on the boundary layer separation point. Namely, an increase in the injection velocity postpones the separation of the boundary layer.

We should also note that there were minor difficulties in numerical solution of the system (27). Namely, the program stopped working for some input values of the parameter of the form f_1. However, the problems were faced only for positive values of the parameter f_1, while for negative values the program ran uninterruptedly.

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